Pareto-Rational Verification

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Joint work with

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Highlights ’22

Formal Verification

Motivation: ensure the correctness of systems responsible for **critical tasks**

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Limitations:

- check **single behavior** of the system
- against **potentially irrational behaviors** of environment

The Model

Stackelberg-Pareto game (SP game) [BRT21]: $\mathcal{G} = (G, \Omega_0, \Omega_1, \ldots, \Omega_t)$

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Payoff of ρ for Player 1 is the **vector of Booleans** $\text{pay}(\rho) \in \{0, 1\}^t$

$\Omega_1 = \text{Inf}(\{v_6\})$

$\Omega_2 = \text{Inf}(\{v_3\})$

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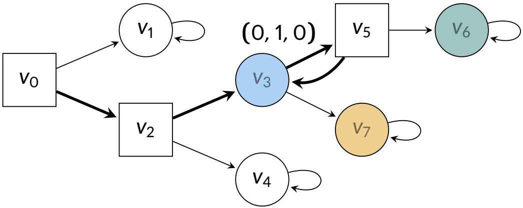

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- **order** $\leq$ on payoffs, e.g., $(0, 1, 0) < (0, 1, 1)$

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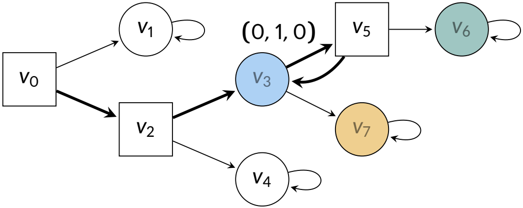
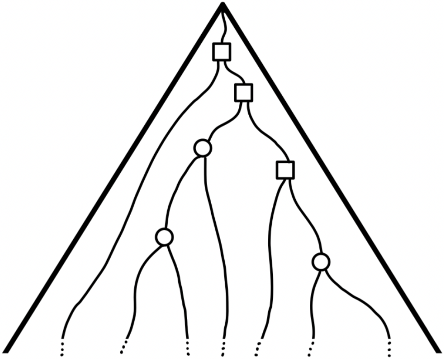


What Do We Verify?

Behavior of the system: **finite-memory strategy** for Player 0



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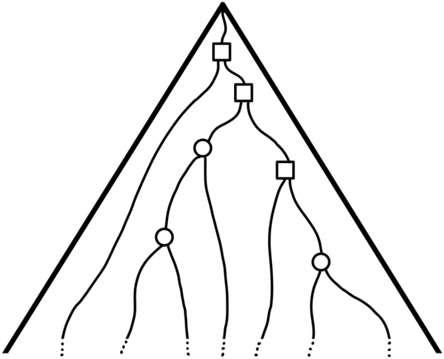

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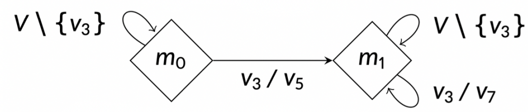
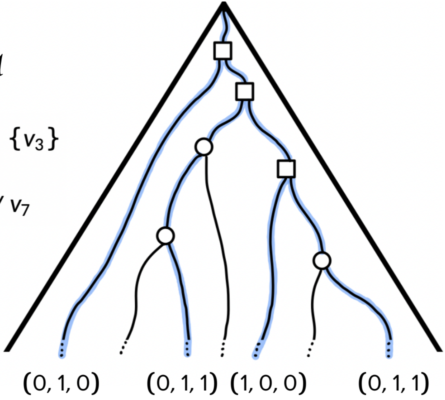
Deterministic Moore machine $\mathcal{M}$



Set of **consistent plays**: Plays_{σ_0}



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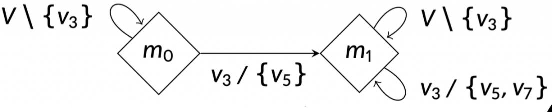
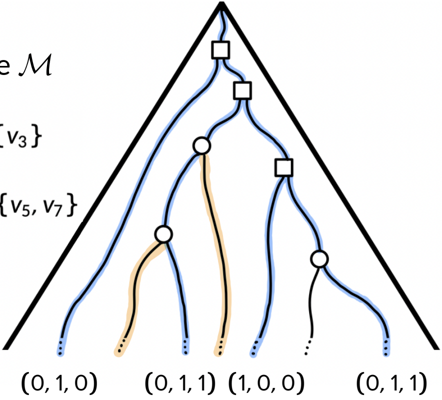
Nondeterministic Moore machine $\mathcal{M}$



Embeds a **set of strategies** $[\![\mathcal{M}]\!]$



Universal Pareto-Rational Verification Problem (UPRV problem)

Decide whether for all strategies $\sigma_0 \in [\![\mathcal{M}]\!]$ of Player 0, every play $\rho \in \mathsf{Plays}_{\sigma_0}$ with $\mathsf{pay}(\rho) \in P_{\sigma_0}$ are such that $\rho \in \Omega_0$

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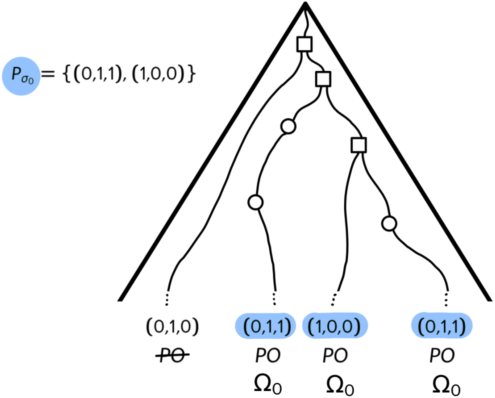


Complexity Results

Study both problems for **parity**, **Boolean Büchi**, and **LTL** objectives

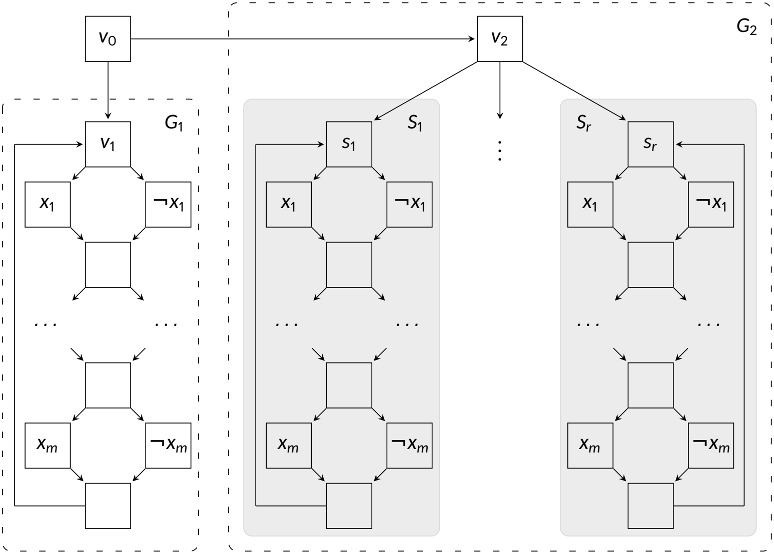
PRV Problem

Objective	Complexity class
Parity	co-NP-complete
Boolean Büchi	Π_2P-complete
LTL	PSPACE-complete

UPRV Problem

Objective	Complexity class
Parity	PSPACE, NP-hard, co-NP-hard
Boolean Büchi	PSPACE-complete
LTL	2EXPTIME-complete

Cool Reductions: co3SAT, Σ_2QBF, solving games, ...



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PRV and UPRV problem

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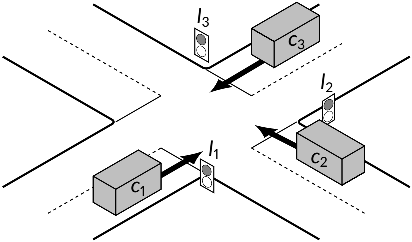

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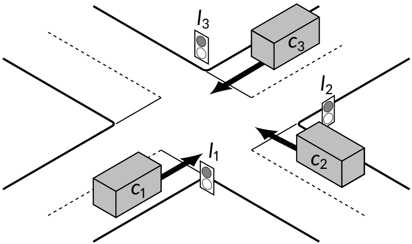

Thank you!

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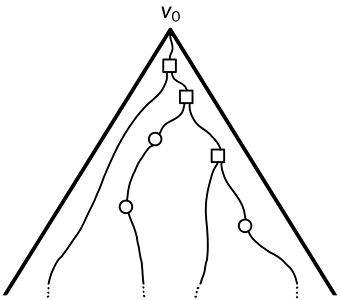

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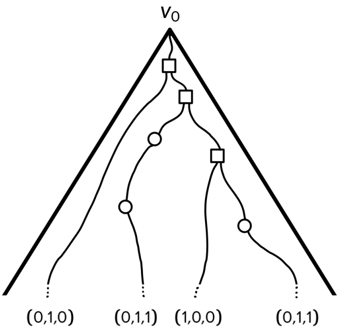

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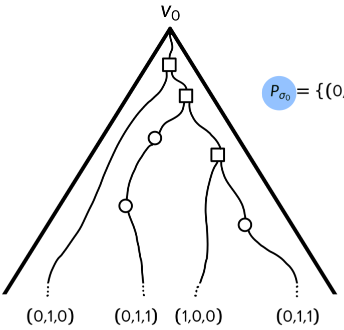
   - corresponding **set of payoffs** $\{\text{pay}(\rho) \mid \rho \in \text{Plays}_{\sigma_0}\}$
   - identify **Pareto-optimal** (PO) payoffs (maximal w.r.t. $\leq$) : set P_{σ_0}



$P_{\sigma_0} = \{(0,1,1), (1,0,0)\}$