Stackelberg-Pareto Synthesis

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Joint work with

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Journées GT Vérif

Outline

1. Reactive Synthesis
2. Stackelberg-Pareto Synthesis
3. Our Results

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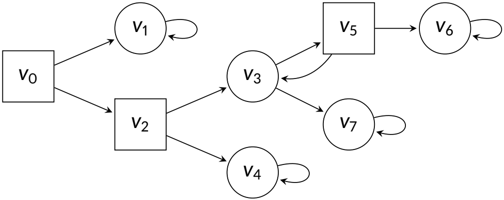
Games: Arenas, Plays and Objectives

Game Arena: tuple $G = (V, V_0, V_1, E, v_0)$ with (V, E) a directed graph



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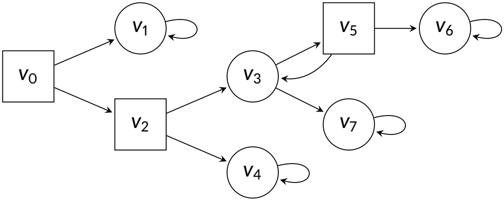

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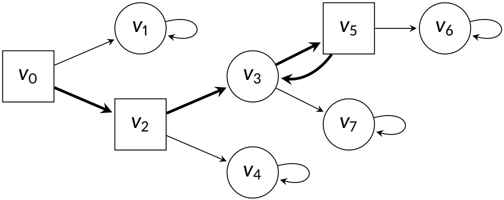

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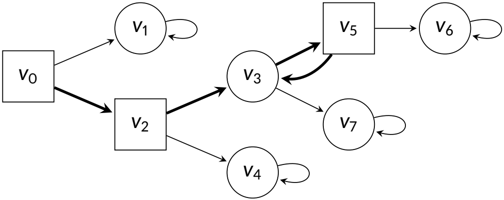

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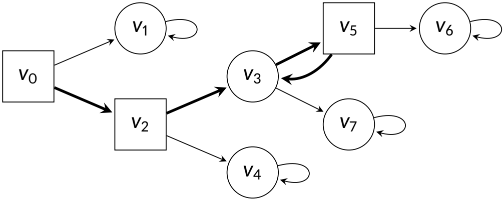

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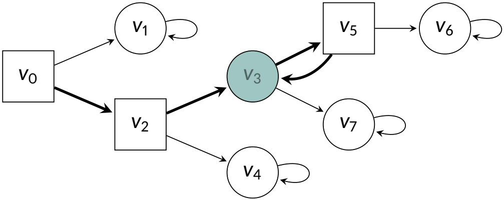

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- **reachability**: plays which *visit* **T** $\subseteq V$

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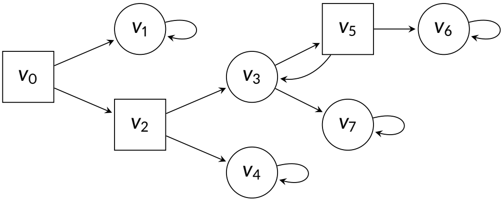
Strategy $\sigma_i : V^* \times V_i \rightarrow V$ **dictates the choices** of Player i



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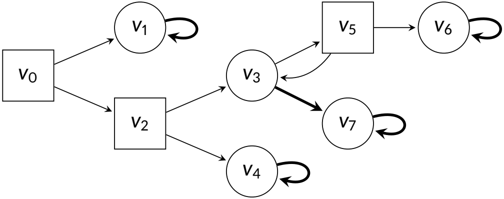

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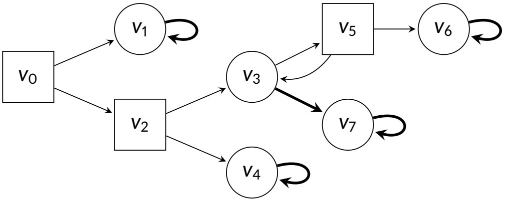

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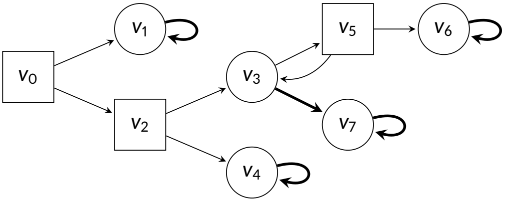

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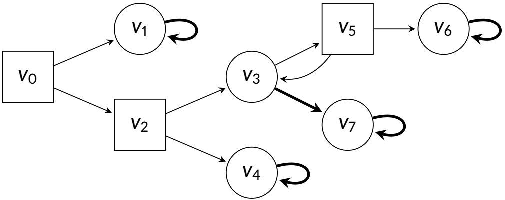

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$\to \text{Plays}_{\sigma_0} = \{v_0 v_1{}^\omega, v_0 v_2 v_4{}^\omega, v_0 v_2 v_3 v_7{}^\omega\}$

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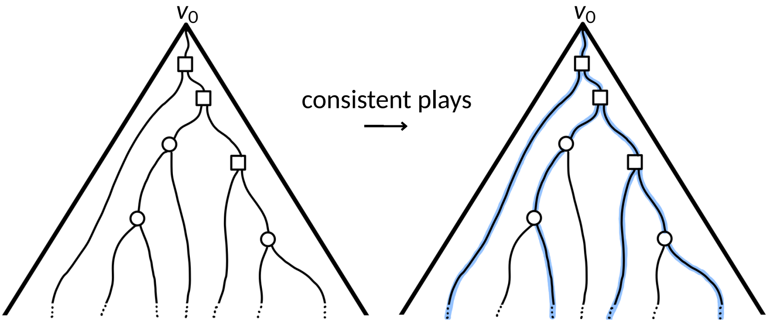

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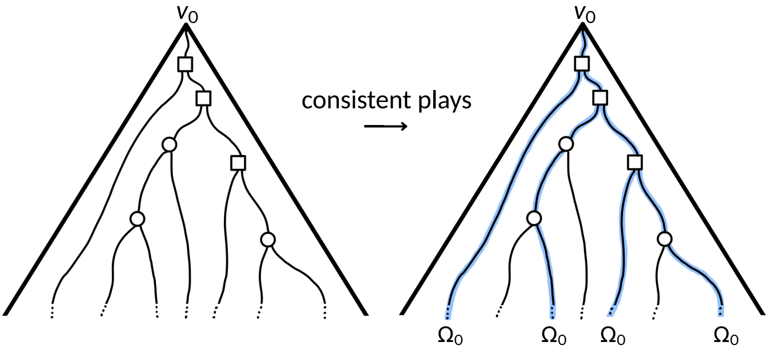

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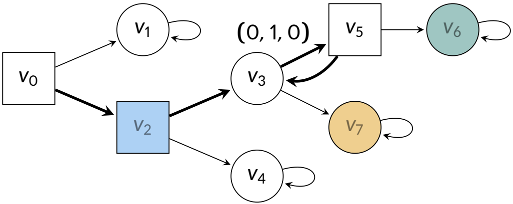

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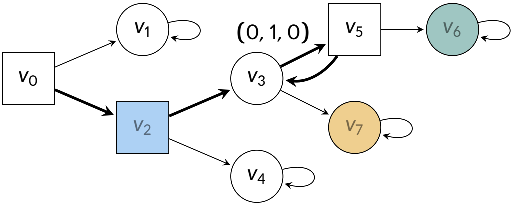

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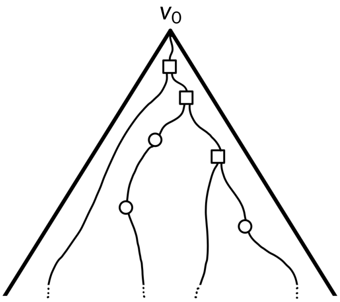

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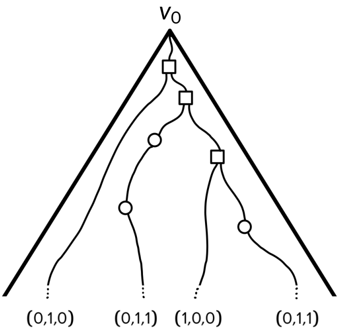

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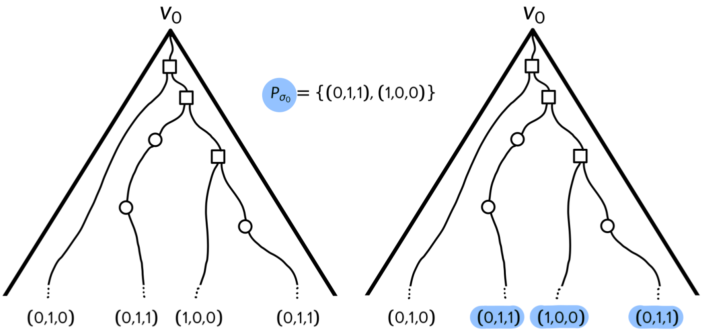

Stackelberg-Pareto Synthesis Problem

Stackelberg-Pareto Synthesis Problem (SPS problem)

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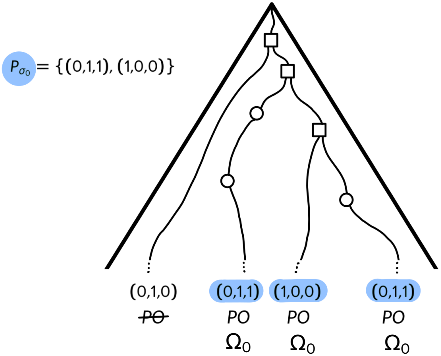

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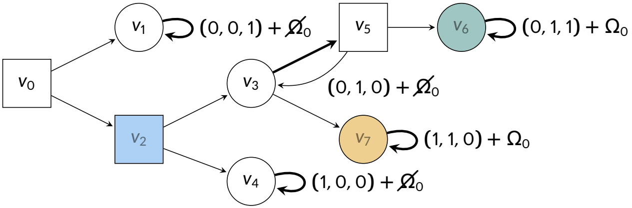
Consider σ_0 such that $\sigma_0(v_3) = v_5$



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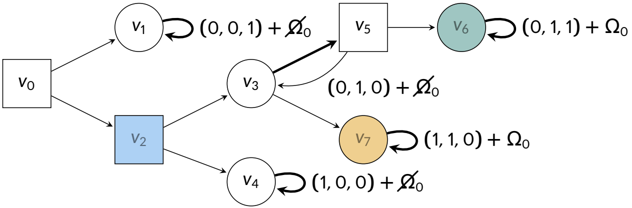

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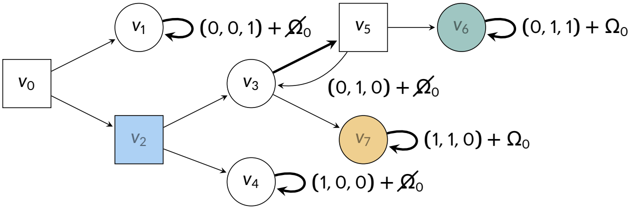

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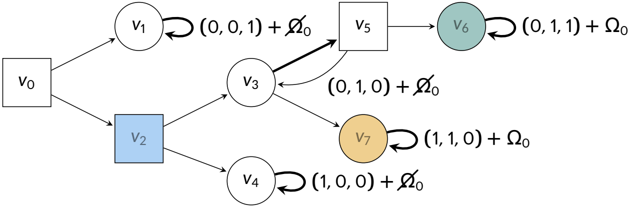

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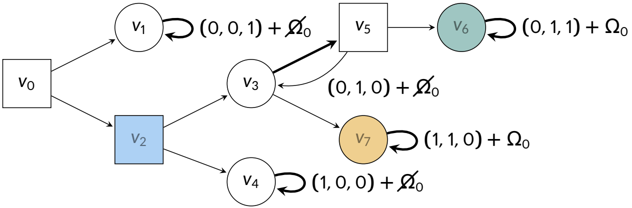

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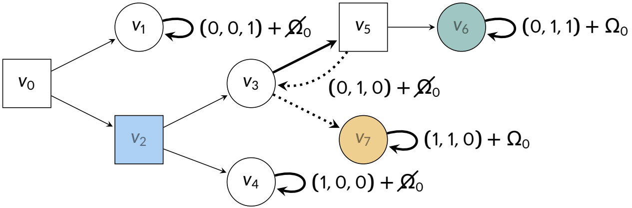
Strategy σ_0 is **not a solution** to the SPS problem, e.g., $\rho = v_0 v_2 (v_4)^\omega \notin \Omega_0$
$\longrightarrow$ the only other **memoryless** strategy is **not a solution either**

SPS Problem Example (2/2)

Finite-memory strategy σ_0' s.t. $\sigma_0'(v_0v_2v_3) = v_5$ and $\sigma_0'(v_0v_2v_3v_5v_3) = v_7$



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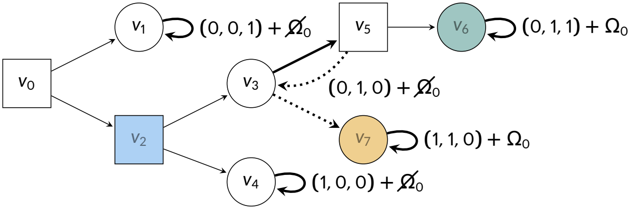

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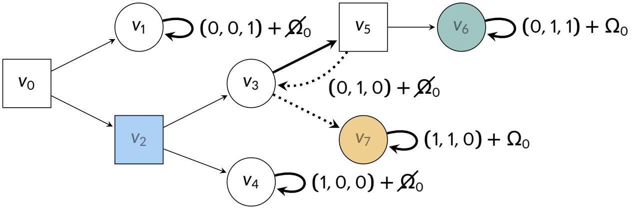

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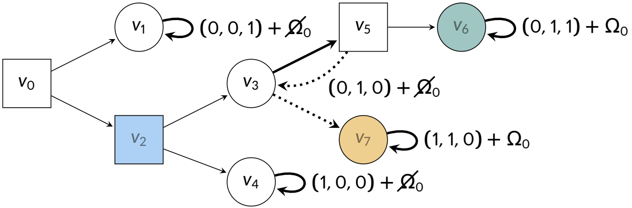

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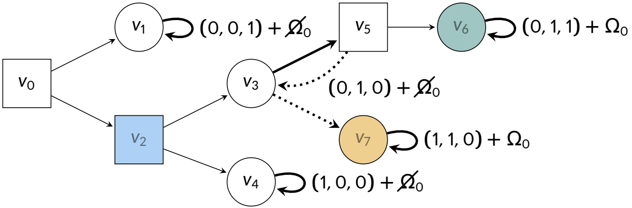

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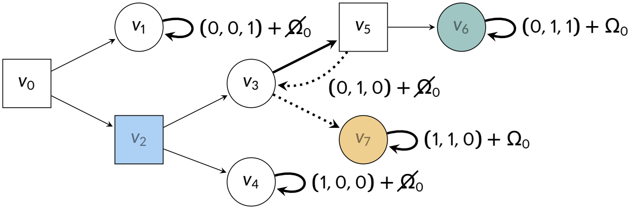

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- parity (**parity SP games**): models general class of ω-regular objectives
- reachability (**reachability SP games**): simpler setting

Outline

1. Reactive Synthesis
2. Stackelberg-Pareto Synthesis
3. Our Results

Our Results on SP games

NEXPTIME-Completeness of the SPS problem

The SPS problem is NEXPTIME-complete for reachability SP games and for parity SP games

Fixed-Parameter Complexity of the SPS problem

Solving the SPS problem is FPT for reachability SP games for parameter t (number of objectives of Player 1) and FPT for parity SP games for parameters t and the maximal priority according to each parity objective of Player 1

Sound: in practice, we can assume those parameters to have **small values**

NEXPTIME algorithm **not** FPT & FPT algorithm **not usable** for membership

Complexity Class

NEXPTIME-Membership

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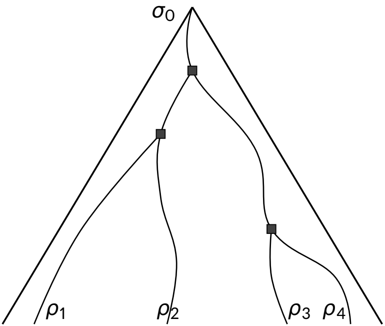
Start from a solution σ_0 to the SPS problem and **one play** ρ_i **per PO payoff**



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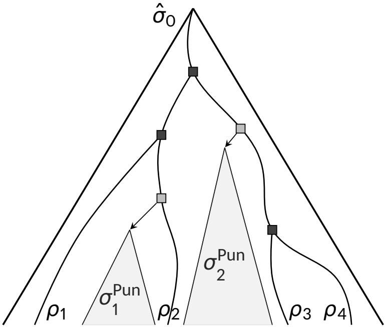

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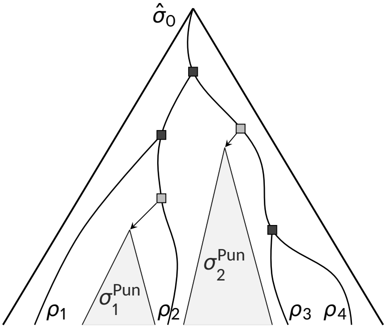

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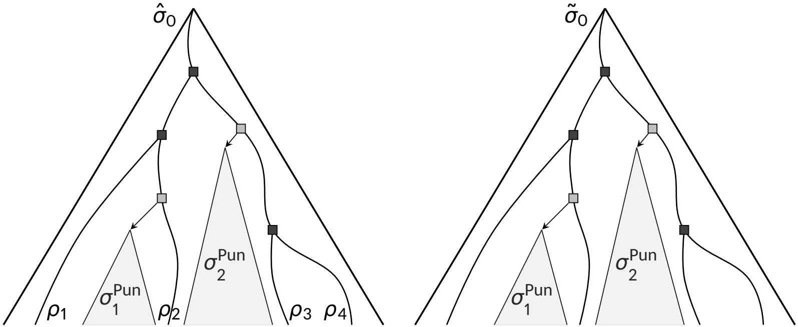

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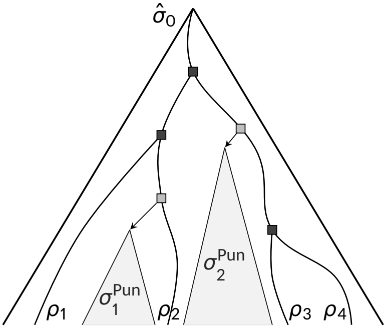

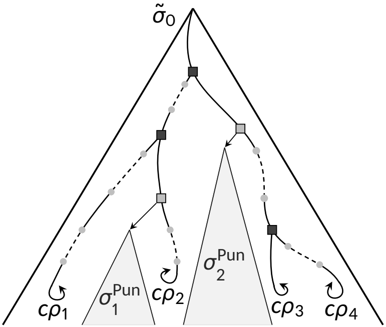

The SPS problem is NP-hard on Tree Arenas

Simple setting of **tree arenas**: trees with loops on leaves

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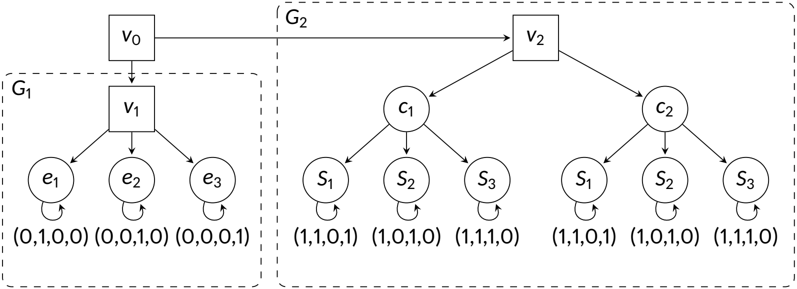

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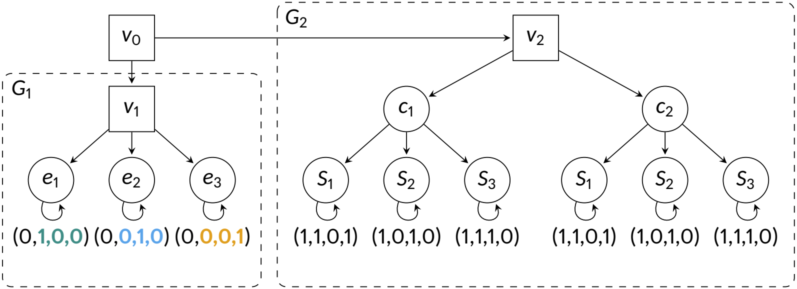

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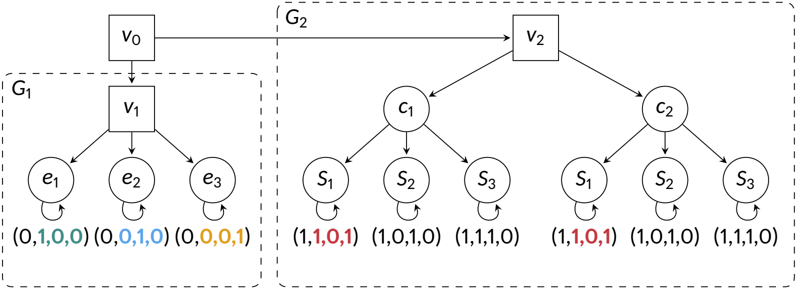

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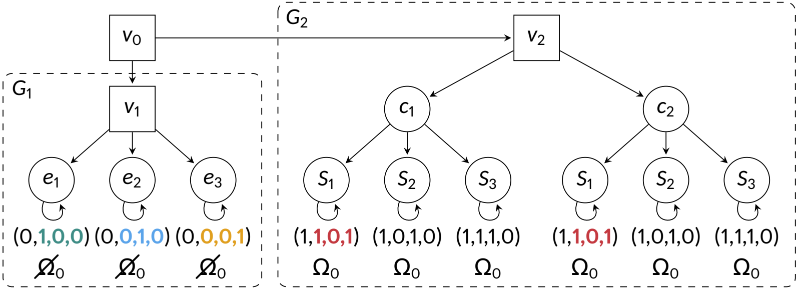

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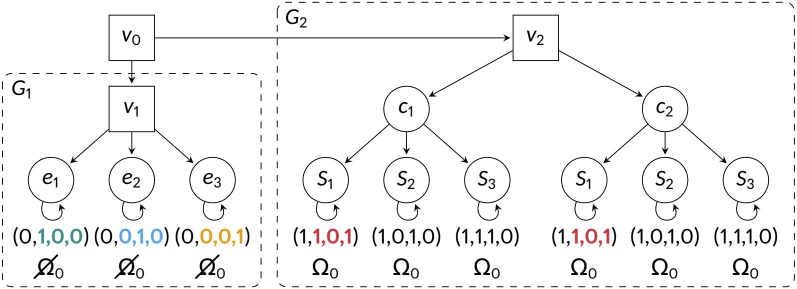

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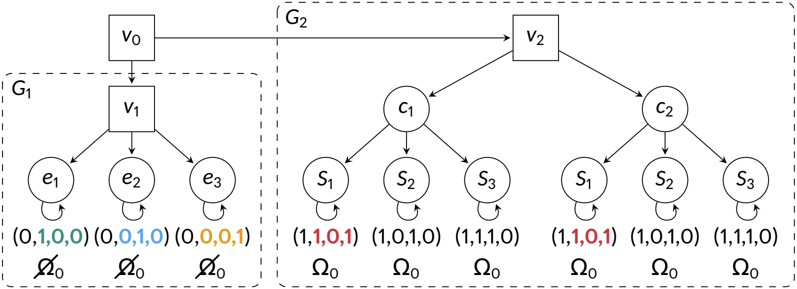

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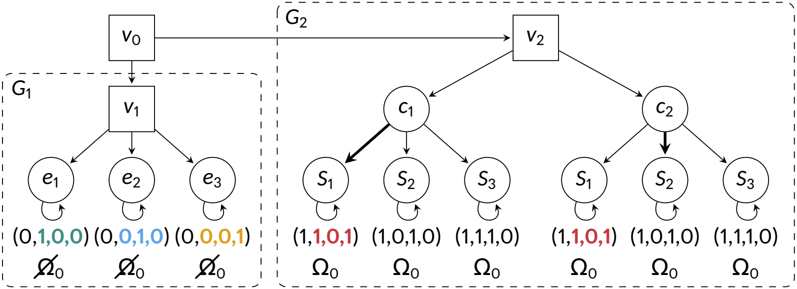

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Hardness

NEXPTIME-Hardness

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Intuition: use **succinct variant** of Set Cover problem (NEXPTIME-complete)
$\longrightarrow$ Set Cover problem succinctly defined using **CNF formulas**

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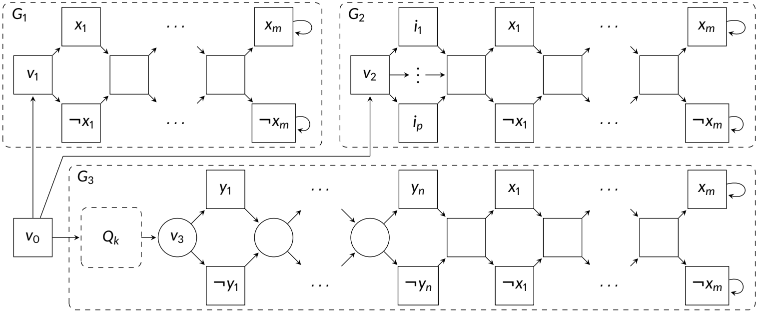

Challenger-Prover Game

To show FPT results: reduction to **Challenger-Prover game** (C-P game)

- **two-player zero-sum game** $\mathcal{G}'$, created from $\mathcal{G}$
- played between **Challenger** ($\mathcal{C}$) and **Prover** ($\mathcal{P}$)
- **solution** to the SPS problem in $\mathcal{G}$ $\iff$ **winning strategy** for $\mathcal{P}$ in $\mathcal{G}'$
- described in a **generic way**, later **adapted** to parity/reachability

Intuition: $\mathcal{P}$ tries to **show** the existence of a solution, $\mathcal{C}$ tries to **disprove** it

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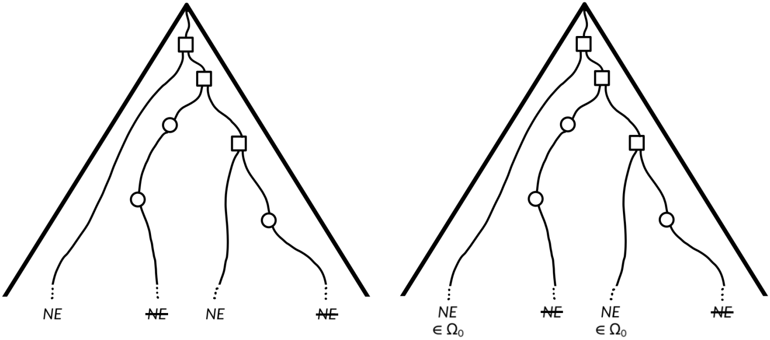
Adversarial Rational Synthesis [FKL10, KPV16]

- **multiplayer** game
- Player 0 = system, Players 1 to n = **components** of environment
- **rationality**: Players 1 to n settle to a **Nash Equilibrium** (NE), given σ_0

⟶ Player 0 **must satisfy** his objective when the environment plays **any** NE



Setbacks: components are independent selfish individuals, no cooperation

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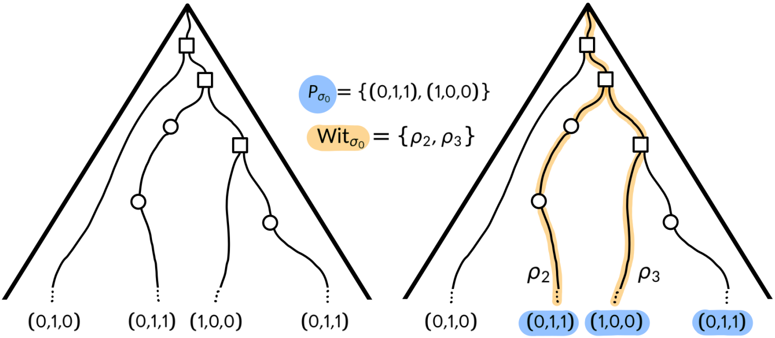
Witnesses

C-P game uses important notion of **witness**

- given σ_0, we have the set P_{σ_0} of **PO payoffs**
- for each $p \in P_{\sigma_0}$, **there exists** ρ s.t. $\text{pay}(\rho) = p$
- select **one such** ρ for each $p \in P_{\sigma_0}$ (**witness of** p): set Wit_{σ_0}

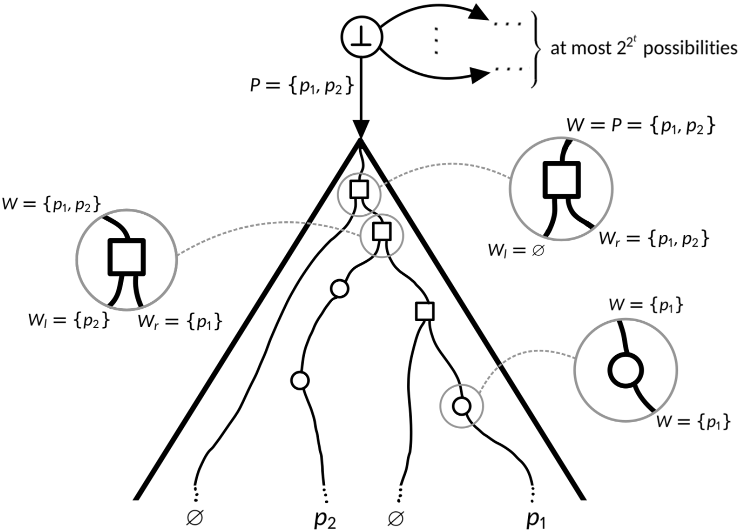


Intuition on the C-P game

w.l.o.g. we consider SP games s.t. each vertex has at most **two successors**

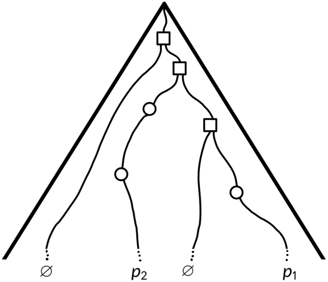
1. $\mathcal{P}$ **selects a set** P of payoffs, he announces it is P_{σ_0} for σ_0 he is building
2. $\mathcal{P}$ tries to show the existence of a **set of witnesses for** P
3. After selection, **one-to-one correspondence** between plays in $\mathcal{G}$ and $\mathcal{G}'$
   - vertices in $\mathcal{G}'$ are **augmented with a set** W which is a subset of P
   - initially $W = P$
   - after history in $\mathcal{G}'$, W **contains** p if the **corresponding history** in $\mathcal{G}$ is **prefix of the witness for** p in the set Wit_{σ_0} that $\mathcal{P}$ is building

Witnesses in the C-P Game



Objective in the C-P Game

Given a play ρ' in $\mathcal{G}'$, there is a corresponding play ρ in $\mathcal{G}$



If play ρ guessed to have payoff p **(1)**

- check that $\text{pay}(\rho) = p$
- check that $\rho \in \Omega_0$

Otherwise

- if $\text{pay}(\rho) = p \in P$, check that $\rho \in \Omega_0$ **(2)**
- else check $\text{pay}(\rho) < p \in P$ **(3)**

Reachability SP game: augment the arena with **set of satisfied objectives**
$\longrightarrow$ checking (1-3) = **Büchi objective**

Parity SP game: checking (1-3) = **Boolean combination of Büchi objectives**

C-P Game for our Running Example