Stackelberg-Pareto Synthesis

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Joint work with
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Highlights ’21

Reactive Synthesis

Reactive systems: systems which constantly interact with the environment

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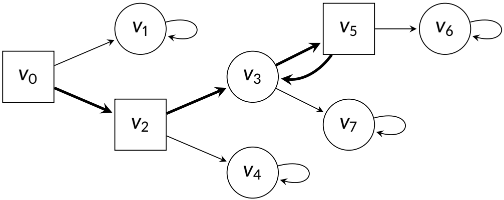
Classical approach for RS: two-player games played on graphs [GTW02]



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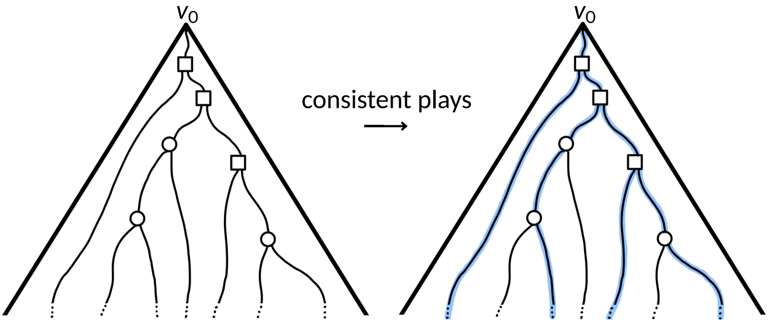

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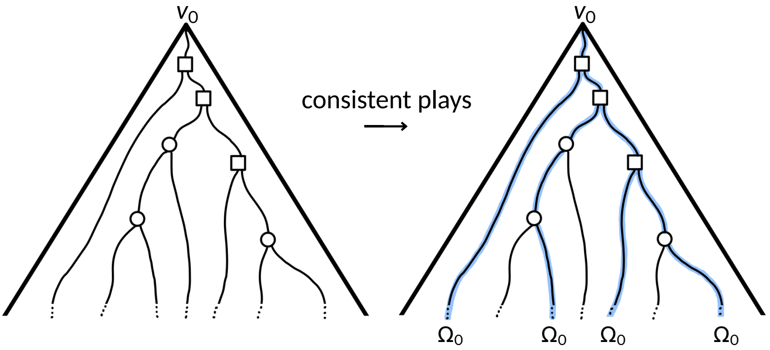

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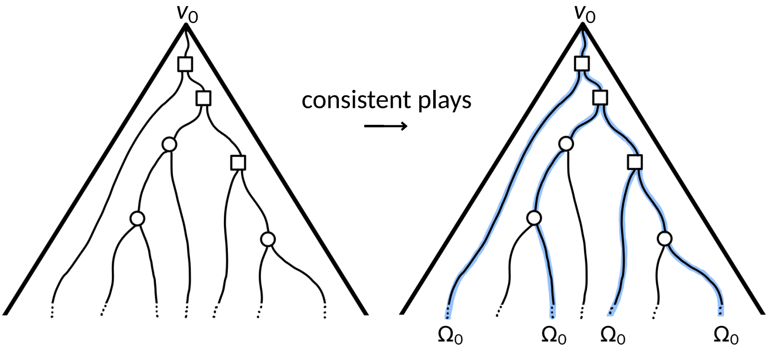

Bold abstraction of reality: **only goal** of environment = **make system fail**

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Alternative: framework of **Stackelberg games** [vS37] (non-zero-sum)

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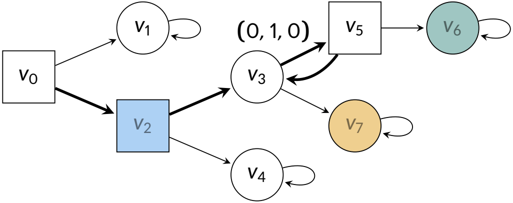

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- **order** $\leq$ on payoffs, e.g., $(0, 1, 0) < (0, 1, 1)$

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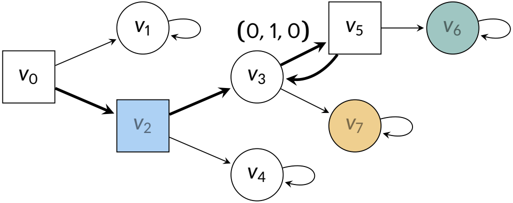

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Stackelberg-Pareto Synthesis Problem (SPS problem)

The SPS problem is to decide whether there exists a strategy σ_0 for Player 0 such that for every play $\rho \in \text{Plays}_{\sigma_0}$ with $\text{pay}(\rho) \in P_{\sigma_0}$, it holds that $\rho \in \Omega_0$

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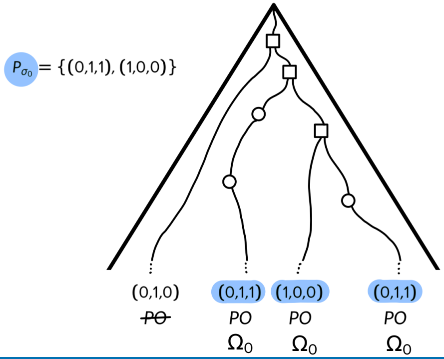
→ Player 0 must **satisfy** Ω_0 in every such **rational response**



Our Results on SP games [BRT21]

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Solving the SPS problem is FPT for reachability SP games for parameter t (number of objectives of Player 1) and FPT for parity SP games for parameters t and the maximal priority according to each parity objective of Player 1

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Thank you !

Bibliography

[BRT21] Véronique Bruyère, Jean-François Raskin, and Clément Tamines.
Stackelberg-pareto synthesis.
In Serge Haddad and Daniele Varacca, editors, *32nd International Conference on Concurrency Theory, CONCUR 2021, August 24-27, 2021, Virtual Conference*, volume 203 of *LIPIcs*, pages 27:1–27:17. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2021.

[GTW02] Erich Grädel, Wolfgang Thomas, and Thomas Wilke, editors.
Automata, Logics, and Infinite Games: A Guide to Current Research [outcome of a Dagstuhl seminar, February 2001], volume 2500 of *Lecture Notes in Computer Science*. Springer, 2002.

[vS37] Heinrich Freiherr von Stackelberg.
Marktform und Gleichgewicht.
Wien und Berlin, J. Springer, 1937.

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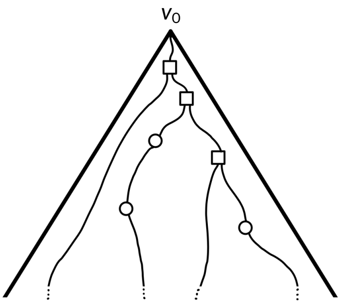

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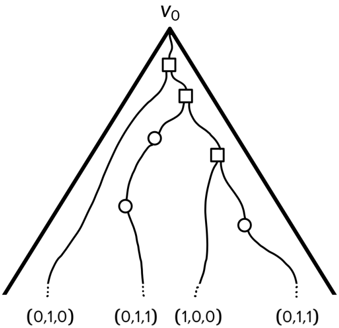

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   - corresponding **set of payoffs** $\{\text{pay}(\rho) \mid \rho \in \text{Plays}_{\sigma_0}\}$
   - identify **Pareto-optimal** (PO) payoffs (maximal w.r.t. $\leq$) : set P_{σ_0}